Int. J. Heat Mass Transfer. Vol. 14, pp. 1213–1216. Pergamon Press 1971. Printed in Great Britain

SHORTER COMMUNICATIONS

FREE CONVECTIVE HEAT TRANSFER ON A VERTICAL CYLINDER WITH CONCENTRATION GRADIENTS

J. L. KYLE* and P. F. DICKSON
Colorado School of Mines, Golden, Colorado, U.S.A.

(*Received* 28 *April* 1970 *and in revised form* 6 *November* 1970)

NOMENCLATURE

A_c, area of cylinder based on log-mean radius corresponding to $(r_2 - r_1)$ [ft^2];
A_s, surface area of cylinder [ft^2];
B, coefficient on mass transfer Grashof number;
C_p, heat capacity [Btu/lb-°F];
g, gravitational acceleration [ft/s^2];
N_{Gr_z}, Grashof number, $g\beta\theta_w z^3/\nu^2$;
$N_{Gr_z^*}$, mass-transfer Grashof number, $g\varepsilon\psi z^3/\nu^2$;
h, heat transfer coefficient [$Btu/ft^2 h$ °F];
k, thermal conductivity [Btu/ft h °F];
N_{Nu_z}, Nusselt number, hz/k;
p^*, partial pressure of the diffusing vapor [mm Hg];
P_t, total pressure [mm Hg];
N_{Pr}, Prandtl number, $C_p\mu/k$;
Q, heat flux [Btu/h];
r, thermowell radius [in.];
T, thermowell temperature [°F];
T_a, ambient-air temperature [°F];
T_r, boundary-layer reference temperature, $T_s - 0{\cdot}38 (T_s - T_a)$ [°F];
T_s, cylinder-surface temperature [°F];
X_{Ao}, mole fraction of diffusing vapor at the liquid interface;
X_A, mole fraction of diffusing vapor in the free stream $= p^*/P_t$;
z, axial distance from lower edge of cylinder [ft].

Greek letters
β, $-(1/\rho)(\partial\rho/\partial T)_{p,X}$ [$°R^{-1}$];
θ, $T_s - T_a$ [°R];
μ, viscosity [lb/ft s];
ν, kinematic viscosity, μ/ρ [ft^2/s];
ε, $-(1/\rho)(\partial\rho/\partial X)_{p,T}$;
ψ, $X_{Ao} = X_A$;
ρ, density [lb/ft^3].

* Presently with Union Oil Company.

FREE convection adjacent to surfaces has received considerable attention in the literature. Eckert and Jackson [1] and Eckert and Drake [2] used von Karman's integral method to obtain expressions for the heat transfer coefficient for a vertical, flat plate with turbulent and laminar boundary layers, respectively. An evaluation of the range of applicability and accuracy of integral methods was given by Levy [3]. Comparisons were also made by Millsaps and Pohlhausen [4] and Eckert and Jackson [1]. Conclusions were that reasonable results could be attained using integral methods where exact analytical solutions are difficult.

Dickson and Traxler [5] extended the theoretical work of Eckert *et al.* [1, 2] to include simultaneous mass transfer from a liquid interface located directly below a heated plate. Integral solutions are known to be fairly insensitive to the profile chosen in giving the correct powers on dimensionless groups. However, coefficients can be widely affected by profile selections. An experimental program was undertaken to verify the analytical results of Dickson and Traxler [5] in predicting the enhancing of retarding effects of mass transfer on the heat transfer coefficient.

EXPERIMENTAL WORK

Design criteria

To study laminar, free-convective heat transfer, a method was used which allowed simultaneous determination of heat flux and the metal–fluid interface temperature. Thermocouples inserted into the cylinder wall at three radial locations provided measurement of the heat flux while extrapolation of these temperatures to the cylinder wall gave the interface temperature.

Using Fourier's Law for conductive heat transfer in a cylinder and Newton's Law of Cooling (the defining equation for h) for heat transfer at a solid–fluid interface, the following equation can be written:

$$Q = \frac{kA_c(T_1 - T_2)}{r_2 - r_1} = hA_s(T_s - T_a).$$

Solving for h is now a matter of rearrangement:

$$h = \frac{kA_c(T_1 - T_2)}{A_s(r_2 - r_1)(T_s - T_a)}.$$

Experimental mass transfer was accomplished using either distilled water or 1-4 dioxan (Eastman, P-2144) as the diffusing vapor. The water vapor enhanced free convection while the dioxane had a retarding effect on free convection.

To insure negligible radiative heat transfer effects, the outer surface of the cylinder was covered with a layer of aluminum foil. The emittance of aluminum foil is approximately 0·087 at 212°F [6] as compared to about 0·50 at 450°F for oxidized stainless steel [7]. The resistance of the foil layer was found to be negligible by comparing experimentally determined heat transfer coefficients with those predicted from [2].

Equipment

A type 304 stainless steel cylinder 8 in. long by $1\frac{3}{16}$ in. i.d. by 4 in. o.d. was used for heat conduction. Heat was generated by a Chromolox, stainless steel, 500 W, platen heater 8 in. long by 1 in. o.d. within the stainless steel cylinder (Fig. 1). An aluminum sleeve 8 in. long by 1 in. i.d. by $1\frac{3}{16}$ in. o.d. was inserted between the platen heater and the stainless steel cylinder to provide a more even heat distribution from the heater to the cylinder. To insure that convective flows would be only those arising because of the experimental system, the composite cylinder was suspended in a plywood enclosure, open at the top and 72 in. high by 40 in. wide by 30 in. deep.

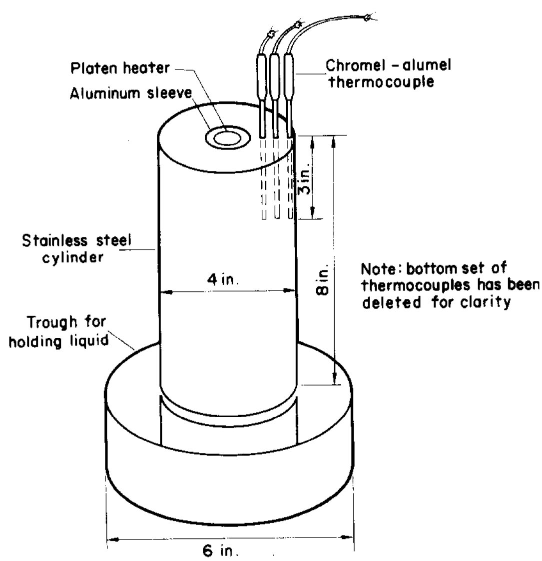

FIG. 1. Diagram of experimental apparatus.

A set of three $\frac{1}{16}$-in. dia. thermowells was drilled into each end of the stainless steel cylinder. One thermowell in each set was located at $\frac{17}{16}$ in., $\frac{23}{16}$ in. and $\frac{29}{16}$ in. from the center of the platen heater. The top and bottom sets were drilled to a depth of 3 and 2 in., respectively.

Temperature measurements were made using 6 Thermo-Electric, sheathed, chromel–alumel, minature, mineral insulated thermocouples 6 in. long by $\frac{1}{16}$ in. o.d. A flexible metal covering to which each thermocouple was grounded also protected the extension wire. Millivolt output from the thermocouples was recorded on a Honeywell Electronik 19 dual-channel, strip-chart recorder. A four-deck, 6-position, silver-contact, rotary switch provided a choice of 6 temperature difference measurements. Two 24-gauge, chromel alumel thermocouples were used to measure ambient-air temperature and liquid-interface temperature.

Liquid for the mass-transfer experiments was held in a circular trough 4 in. i.d. by 6 in. o.d. located directly below the cylinder. This liquid could be heated by 25 in. of $\frac{1}{4}$-in. copper tubing containing 50 in. of nichrome ribbon rated at 3·45 Ω per ft.

Procedure

Procedure with or without liquid evaporation varied little. The platen-heater voltage was adjusted to a desired value, after which 6–8 h were required for the cylinder to reach a steady-state temperature. If a liquid was used in the run, the nichrome-ribbon voltage was adjusted to a desired value several hours before the cylinder reached its steady-state temperature.

The rotary switch was turned to position 1, and the temperature of the inner, top thermowell was recorded. The cylinder was assumed to be at a steady-state temperature if the temperature reading remained constant over a five-min interval. The five remaining temperature differences, referred to this first thermocouple, were then recorded several times during each run. At the same time, ambient-air temperature and, if applicable, liquid-interface temperature were recorded.

Finally, the inner, top thermowell temperature was again recorded as a check to insure that the cylinder had remained at the steady-state temperature during the entire run. In addition, the ambient wet-bulb temperature and the barometric pressure were measured following each run.

DISCUSSION OF RESULTS

Forty-two experimental runs were made in three cylinder temperature ranges as measured by the inner, top thermocouple: 195–202°F, 392–411°F and 549–591°. Fifteen of the experiments were conducted in the absence of mass transfer. Diffusing water and dioxane vapors were used in 22 and 8 of the remaining runs, respectively. Liquid water temperatures were maintained in three temperature ranges (68–81°F, 121–130°F and 141–151°F) as were the liquid dioxane temperatures (84–91°F, 142–147°F and 174°F).

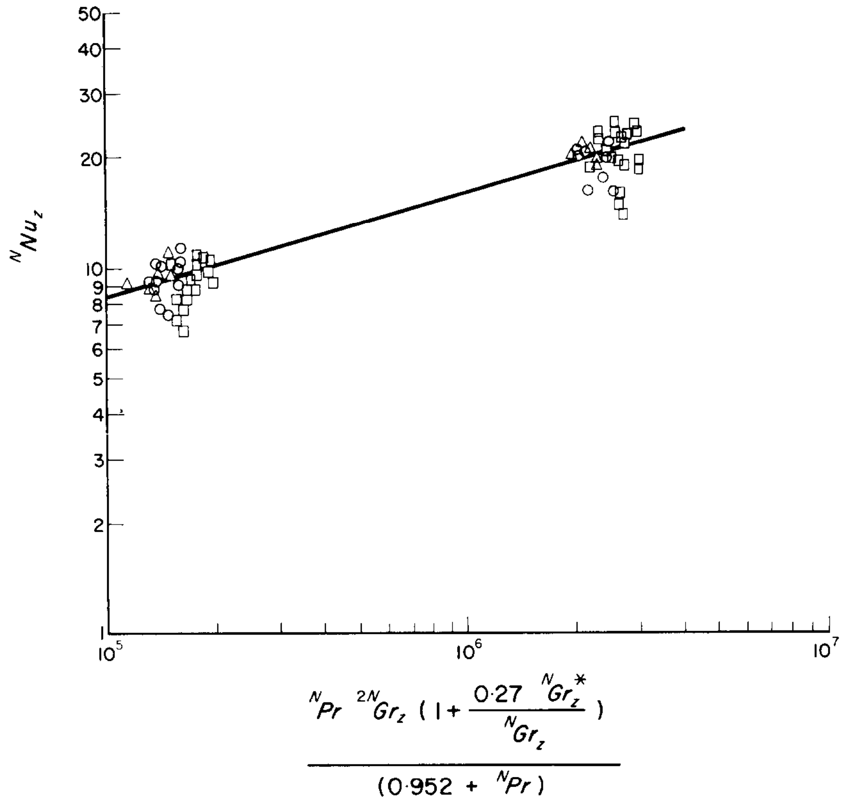

FIG. 2. Local Nusselt number correlation. (Note: Due to congestion of points, all data in the more dense regions are not shown.)

○ no mass transfer; □ water vapor as diffusing component; △ 1-4 dioxane vapor as diffusing component.

The maximum Grashof number reached in the experimental work was 1.3×10^7 which lies within the laminar flow region (up to 1×10^8). According to Dickson and Traxler [5] the expression relating simultaneous heat and mass transfer in laminar free convection is:

$$N_{Nu_x} = \frac{0.508(N_{Pr}^{1/2})(N_{Gr_x} + 0.25\, N_{Gr_x^*})^{1/4}}{(0.952 + N_{Pr})^{1/4}}. \quad (1)$$

In the absence of mass transfer this equation reduces to that of Eckert and Drake [2].

Nusselt number was plotted on the ordinate vs. the following term:

$$\frac{(N_{Pr}^{2})(N_{Gr_x})\,[1 + (BN_{Gr_x^*}/N_{Gr_x})]}{(0.952 + N_{Pr})}.$$

A least-squares fit was applied to determine the value of B and the power on this grouping. As shown, the slope of the line was found to be 0.286 rather than the theoretical value of 0.25 and B was 0.27 rather than 0.25 as predicted. The effect of increasing the coefficient to decrease the slope of the line merely spreads out the two groups of data. Several points in each group represent the case of no mass transfer and are unaffected by a change in the coefficient; the points representing the retarding effect on heat transfer would move to the left on the plot, and the points representing the enhancing effects on heat transfer would move to the right.

CONCLUSIONS

The equation derived by Dickson and Traxler [5] appears to correctly predict the effects of mass transfer in determining the local heat transfer coefficient for surface configurations approaching the flat plate. The experimentally determined mass-transfer–Grashof number coefficient was found to be

8 per cent greater than the coefficient predicted in the derivation. This error could easily lie within the experimental results since the plot of log N_{Nu_x} vs.

$$\log N_{Pr}{}^2 N_{Gr_x}(1 + 0{\cdot}27\, N_{Gr_x^{\frac{1}{2}}}/N_{Gr_x})/(0{\cdot}952 + N_{Pr})$$

shows a slight deviation from the predicted power on the group $(N_{Gr_x} + 0{\cdot}27\, N_{Gr_x^{\frac{1}{2}}}/N_{Gr_x})$. The experimental power found was 0·286 as compared to the predicted power of 0·25.

ACKNOWLEDGEMENTS

Acknowledgment is made of the financial assistance provided by the Colorado School of Mines Foundation and for a National Aeronautics and Space Administration Traineeship for one of the authors (J.L.K.).

REFERENCES

1. E. R. G. ECKERT and T. W. JACKSON, Analysis of turbulent free-convection boundary layer on flat plate, NACA Report 1015 (1951).
2. E. R. G. ECKERT and R. M. DRAKE, JR, *Heat and Mass Transfer*, pp. 312–315. McGraw-Hill, New York (1959).
3. S. LEVY, Integral methods in natural-convection flow, *Trans. Am. Soc. Mech. Engrs* 77, 515 (1955).
4. K. MILLSAPS and K. POHLHAUSEN, The laminar free-convective heat transfer from the outer surface of a vertical circular cylinder, *J. Aeronaut. Sci.* **25**, 357 (1958).
5. P. F. DICKSON and J. J. TRAXLER, Free convection on a vertical plate with concentration gradients, *Am. Inst. Aeronaut. Astron. Jl* **3**, 1511 (1965).
6. R. H. PERRY, C. H. CHILTON and S. D. KIRKPATRICK, *Chemical Engineers' Handbook*, pp. 10–36, 4th Ed. McGraw-Hill, New York (1963).
7. E. M. SPARROW and R. D. CESS, *Radiation Heat Transfer*, p. 45. Brooks/Cole, California (1966).

Int. J. Heat Mass Transfer. Vol. 14, pp. 1216–1220. Pergamon Press 1971. Printed in Great Britain

EFFECTIVE CONDUCTIVITY FOR CONDUCTION-RADIATION BY TAYLOR SERIES EXPANSION†

E. E. ANDERSON and R. VISKANTA

Heat Transfer Laboratory, School of Mechanical Engineering, Purdue University, Lafayette, Indiana 47907, U.S.A.

(Received 1 *October* 1970 *and in revised form* 29 *December* 1970)

NOMENCLATURE

$E_{b\nu}$, spectral black body emissive power;
E_n, exponential integral function of order n;
k, thermal conductivity;
k_r, radiative conductivity defined by equation (14);
q_r, radiative flux;
q_w, total heat flux;
T, temperature;
α, spectral dependence of the absorption coefficient;
β, state dependence of the absorption coefficient;
ε, emissivity;
κ_R, Rosseland mean absorption coefficient;
θ, dimensionless temperature, T/T_2;
ρ, reflectivity;
ν, frequency;
τ, optical depth, $\int_0^y \beta(y)\,dy$;
τ_0, optical thickness, $\int_0^l \beta(y)\,dy$.

Superscript
*, dimensionless quantity.

Subscript
i, denotes the ith band;
1, denotes the cool boundary;
2, denotes the hot boundary.

† This work was supported under the Advanced Research Projects Agency Institutional Grant SD–102.

INTRODUCTION

AS A RESULT of intensive study, considerable progress has been achieved over the past decade in understanding energy transfer by combined conduction and radiation in semi-transparent media. Rigorous analyses of radiative transfer characteristically involves either an integral or integrodifferential equation which must be solved numerically as the applicable energy equation, since the radiative flux possesses